

Predicting the root of a musical chord: Theoretical principles and contemporary approaches

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Abstract

The prediction of the root of a musical chord brought greatest difficulties to the western music theory since its beginnings. Numerous theorists dealt in the past 300 years with the essence of chord roots and their significance for the analysis of music and also for music psychology is undisputed today. Therefore, the main problems with the prediction of the root of a chord will be elaborated and further investigated and also different approaches will be presented. In addition, some models, which are intended to illustrate the different approaches to this problem, will be explained.

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Talk

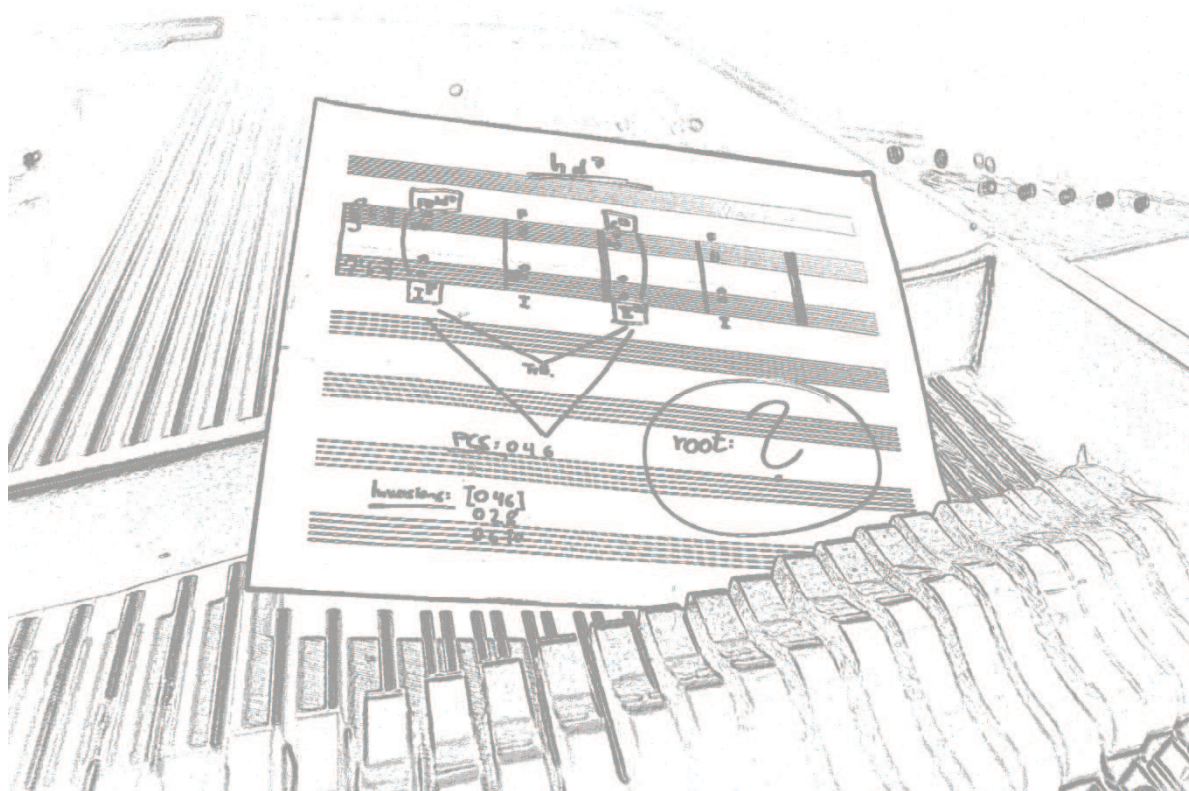


Fig. 1: (Picture: Daniel Öttl).

Dear Professors, students and listeners. First, I would like to thank Prof. Parncutt and the Karl-Franzens-Universität Graz for inviting me to give this talk and I also would like to thank you for coming along to listen – I hope you enjoy it. In this presentation I will consider the current scientific discussion on the theoretical part of "root prediction". I will highlight the problems that occur in root prediction and I'll show three contemporary models for predicting the root of a musical chord. I will examine why many models fail, where the basic problem of root prediction is and in which direction possible solutions could go.

Clarifying the question of the detectability and the identifiability of the root of a chord is a fundamental problem in music analysis, as the perception of chord roots forms the basis of numerous analytical models. But these models cannot provide unambiguous results, as long as these fundamental questions are not cleared up, which is why there is an ongoing and controversial scientific discussion on this topic – especially since the appearance of perceptual psychological approaches in music psychology.

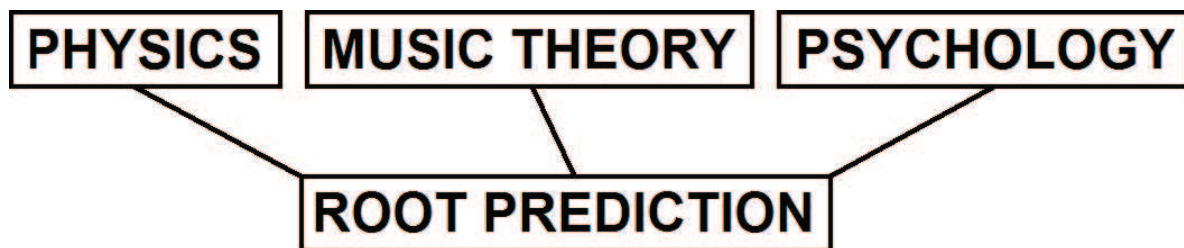


Fig. 2: Most important disciplines which are relevant for the prediction of chord roots.

Associated research projects largely build on the physical principles of music and psychoacoustic approaches in music theory. This is why the basic issue of root prediction must be seen as a key issue of the disciplines of physics, music theory, and psychology.

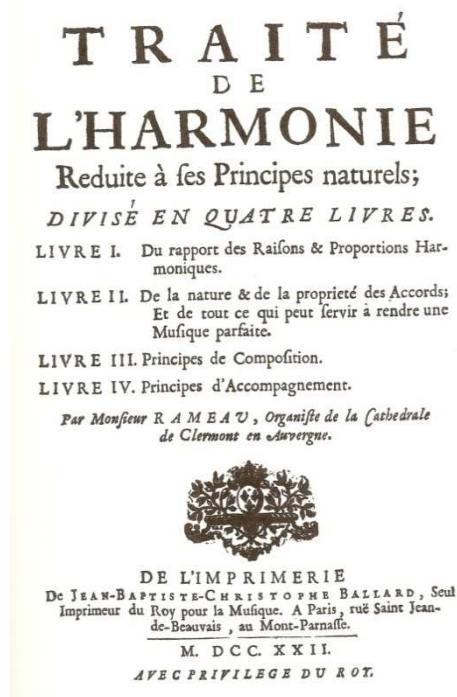


Fig. 3: „Traité de l'harmonie“ (from Rameau, 1971).

The theoretical discussions on the issue of root prediction began with Jean-Philippe Rameau's statements about the root position of chords, the derived inversions and chord extension by additional thirds – written in his seminal work "Traité de l'harmonie" (see Rameau, 1971). There Rameau postulates the principle of stacked thirds which will dominate the theory of root prediction in the following centuries as follows: "[...], we could consider thirds for the time being as the sole elements of all chords. To form the perfect chord, we must add one third to the other; to form all dissonant chords, we must add three or four thirds to one another.

The differences among these dissonant chords arise only from the different positions of these thirds." (Rameau, 1971, p. 39). In the model of stacked thirds, that arose from these basic principles, the tones of a musical chord are (if possible) brought into that position in which they exclusively build thirds as intervals, which is the so-called "root position" (cf. Schmid, 2012, p 35) – as Figure 4 shows with the C-major-dominant-seventh-chord.

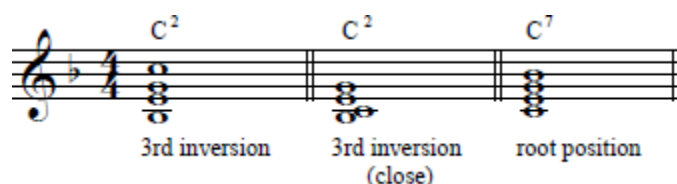


Fig. 4: Predicting the root of a C-major-second chord by stacking thirds.

The first bar shows the third inversion of the C-major-dominant-seventh-chord with a tritone between $\{Bb_3\}$ and $\{E_4\}$ and a perfect fourth between $\{G_4\}$ and $\{C_4\}$. In close layer (in the second bar) appears a major second between $\{Bb_3\}$ and $\{C_4\}$ – this is reason for it is called a second chord. Finally in the third bar the root position is shown – here the whole chord is made of thirds. This model can be complemented by the system of stacked fourths to also show chords in root position that are built of fourth instead of thirds (e.g. $\{C_3 F_3 Bb_4\}$), as Schoenberg does (cf. Schoenberg, 2005, pp. 477 ff.). Consequently, the lowest tone of the root position forms the root of the chord. The principle of stacked thirds itself comes from the major chord, which occurs in the overtone scale: The harmonics 4, 5 and 6 give the major chord – a much bigger problem however is the derivation of the minor chord.



Fig. 5: The appearance of the major chord by the harmonics 4, 5 und 6.

Indeed the minor triad can be seen as mirroring the major triad, for this explanation an undertone scale has to be constructed, which primarily exists in theory but not as a physical reality. Nevertheless, for a long time the principle of stacked thirds appeared a very plausible solution to the matter of predicting the root of a chord. That was because in most instances it predicts for appropriate chords, such as the major and the minor triad, the diminished triad,

the dominant seventh chord etc., that root, which corresponds to the general auditory impression.

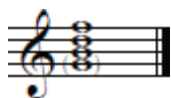


Fig. 6: Incomplete dominant seventh chord.

Although the diminished triad has always been a subject of debate and is sometimes seen as an incomplete dominant seventh chord (i.e. with missing root), which also is getting projected on the dominant ninth chord. But at least since the expansion of tonality and the establishment of the chromatic scale as the basis of tonal music, the principle of stacked thirds is increasingly being called into question, since it is bound to diatonic scales which are shown in the five-line-system of western notation, and thus do not allow the unambiguous prediction of chord roots.

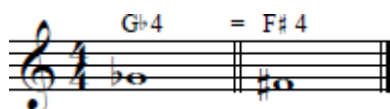


Fig. 7: $G\flat_4$ is enharmonic equivalent with $F\sharp_4$ which occurs with the same frequency.

From a tonal point of view, the enharmonic is irrelevant in equal temperament. So, even the representation of chords cannot be bound to it, because the principle of stacked thirds then leads to different roots for – in a tonal way – equal chords. But this is without a theoretical justification. The following example demonstrates that the principle of stacked thirds (and thus also that one of stacked fourths) can lead only by the enharmonic to entirely different results, although in the equal temperament out of a physical perspective the same tones (i.e. frequencies) sound. Also aesthetic justifications regarding voice leading or chord-chains are untenable, as we see in Figure 8. Here are shown two possibilities to represent the hard-diminished chord – in the first case the root is $\{F\sharp_3\}$, in the second case $\{C_2\}$, but both can be assigned without any difficulty to the root position of F-major (here however in different layers) – the resulting roots form the interval of the tritone.

The image shows a musical score with two staves, treble and bass clef, in 4/4 time. The top staff contains four measures of music. The first measure has a chord labeled F#^{b7} with notes F#4, C5, G4, and Bb4. The second measure has a chord labeled F with notes F4, A4, C5, and F5. The third measure has a chord labeled C^{b7} with notes C4, Eb4, G4, and Bb4. The fourth measure has a chord labeled F with notes F4, A4, C5, and F5. The bottom staff contains four measures. The first measure has a chord labeled #1³ with notes F#3, A3, C4, and F#4. The second measure has a chord labeled I with notes F3, A3, C4, and F4. The third measure has a chord labeled V^b with notes C3, Eb3, G3, and Bb3. The fourth measure has a chord labeled I with notes F3, A3, C4, and F4.

Fig. 8: Two possibilities to represent the hard-diminished chord – in the first case the root is {F#₃}, in the second case {C₂}, but both can be assigned without any difficulty to the root position of F major (here however in different layers) – the resulting roots form the interval of the tritone.

To show chords without the enharmonic is possible with the "pitch-class set theory" of Forte (see Forte, 1973): He bases the representation of chords on the chromatic scale with twelve tones – frequency doublings (e.g. {C₃} and {C₄}) are summarized as one "pitch-class" and common occurring sounds get analyzed as one "pitch-class set" (see *ibid.*). This excludes the enharmonic and intervals can be defined as absolute distances of figures. Based on these assumptions mainly in the 20th century a number of different new models were created to identify the roots of chords – Goldbach describes the most important of these in his article "Modelle der Akkordgrundtonbestimmung" (Goldbach, 2009), which means "Models of chord root prediction". He evaluates them and proves sources of error in all models in a comprehensible way.

An analytical system for the collection of chord structures and chord-chains cannot work without a model for the prediction of chord roots. While the system of stacked thirds held validity for a long time, at least by the time of romanticism (and the concomitant expansion of tonality) the inaccuracy of this system had become evident and many theorists were concerned with this problem.

Already von Helmholtz described the nature of combination tones in his work "Die Lehre von den Tonempfindungen als psychologische Grundlage für die Theorie der Musik" (von Helmholtz, 1913), the English title is "On the Sensations of tone as a physiological basis for the theory of music" – and he applied this knowledge to the chord theory.

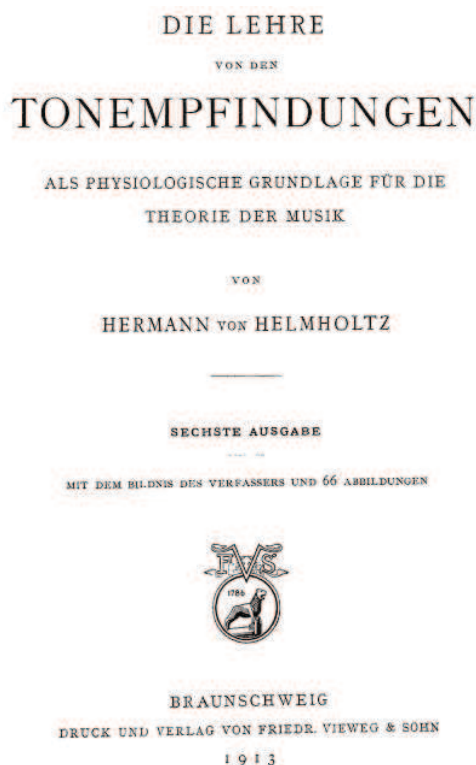


Fig. 9: "Die Lehre von den Tonempfindungen als psychologische Grundlage für die Theorie der Musik" (from von Helmholtz, 1913).

He developed based on the theory of combination tones a new model for root prediction, which he derived from the overtone scale. In the 20th century aside to Arnold Schoenberg, whose theories essentially base on the roman numeral analysis but do not deal directly with the fundamental problem, let alone give solutions, especially Paul Hindemith is to name. He developed a new model for root prediction, which he derived from the overtone scale. Although based on psychoacoustic principles, Ernst Terhardt built on von Helmholtz' determination of combination tones and established the "Theorie der virtuellen Tonhöhe", the theory of virtual pitches (cf. Terhardt, 1998, pp. 360f.). Richard Parncutt continued Terhardts model and also based his theory on differently weighted "root supports" (cf. Parncutt, 1988, p. 65), which he weighted more accurately and structured. Generally speaking, combination tones are created by the apparatus of the human ear and not through audible sound stimulus. Roederer gives as a reason for their existence, if two pure tones with the frequencies f_1 and f_2 sound at once (following Roederer, 2000, p. 45) and he writes relating to the perceptibility of combination tones that it's most easily to percept them, if both tones are different (following *ibid.*). He further explains that the frequency of that combination tone, which is perceptable most clearly with high intensity level, is given by the frequency components: $f_{C1} = f_2 - f_1$ (following Roederer, 2000, p. 46).

More combination tones are perceived at the frequencies $f_{c2} = 2f_1 - f_2$ and $f_{c3} = 3f_1 - f_2$ (according to *ibid.*). According to von Helmholtz, the primary combination tone can make new combination tones with the base sound (following von Helmholtz, 1913, p. 256), which are then second order combination tones (in contrary to first order combination tones) and so on.

In the 20th century aside to Arnold Schoenberg, whose theories essentially base on the roman numeral analysis but do not deal directly with the fundamental problem, let alone give solutions, especially Paul Hindemith is to name. He developed – based on the theory of combination tones – a system for predicting the roots of intervals, making a distinction between "Grundton" (Hindemith, 1940, p. 90), i.e. the root, and "Begleiter" (*ibid.*), i.e. companion. From this he elicits the following interval roots (shown in Figure 10 as filled note heads), although seven of the twelve interval roots are based on sound experiences (marked in Figure 10 with an x).

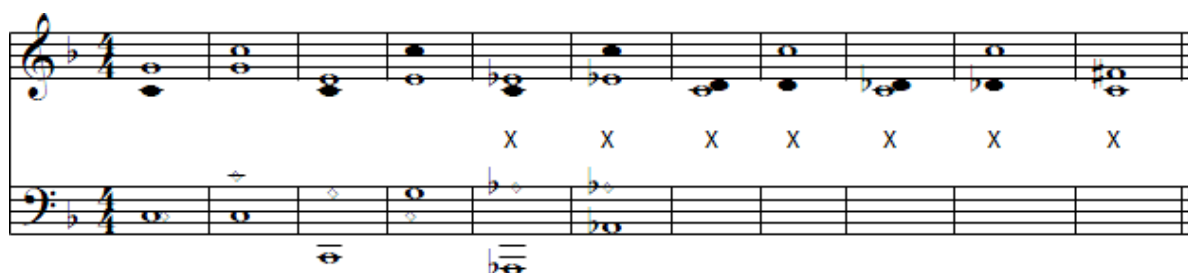


Fig. 10: Interval roots according to Paul Hindemith (Hindemith, 1940, pp. 90 – 106).

About the tritone Hindemith says in general that it has no root (Hindemith, 1940, p. 105), but he had a substitutional root (*ibid.*, p. 106): Even the tone, which reaches the root of the dissolving interval by the smallest step (following *ibid.*, p. 106). Thereby he lays the foundation for the belief, advocated by some later theorists, that the root of a chord (for Hindemith only the tritone) can be determined in some (or in all) cases only in the harmonic context of the sound and in different harmonic contexts different roots can be perceived with the same chord. Hindemith uses this theoretical approach for the prediction of roots and says that we find the root of a chord, if we pick the best interval of the chord (following Hindemith, 1940, p. 120), for which he arranges the intervals in the order shown in Figure 10. But Hindemith's theory remains questionable because it is based on individual intervals in chords

whose supposedly strong emergence he does not prove. Moreover, the results are not always in accordance with the auditory impression, from which he himself starts, as proved impressively by Goldbach (cf. Goldbach, 2009, pp. 391 ff.).

Ernst Terhardt speaks of a hierarchy of different pitch perceptions: He differs spectral pitches (i.e. frequencies, which correspond to parts of the overtone spectrum) from the resulting virtual pitches (Terhardt, 1998, p. 311). Roederer however names spectral pitches as so called "Ohr-Obertöne" (Roederer, 2000, p. 48), or in English "ear overtones", which he defines as frequencies, which are multiples of the original frequency, like $2f_1$, $3f_1$, $4f_1$ etc. (ibid.). So spectral pitches are not physical-acoustic phenomena, but perceptual ones and are therefore not available as a physical sound signal. Virtual pitches are caused by the occurrence of frequencies of the overtone spectrum, at which the lowest (or more) partial (or partials) does (or do) not appear. Nevertheless, the fundamental frequency of the full overtone spectrum is getting perceived and multiple virtual pitches appear (Terhardt, 1998, p. 348). These different perceptions of pitches are processed by the human ear and the resulting (virtual) pitch is what the listener perceives. On this basis Terhardt developed the thesis that our ear analyzes sound objects of every kind (like tones, musical chords, sounds of bells) regarding their relationship to one or more roots (following ibid., p. 398). He leads roots as well as single tones back to virtual pitch phenomena. In doing so he applies the same rules to the prediction of chord roots, as in the determination of virtual pitches – especially as roots are also perceptual phenomena as virtual pitches and actually not tones but pitches (following ibid., p. 398). That means after Terhardt, perceived chord roots do not necessarily have to be a frequency of the original signal, but can be supplemented by the ear. Terhardt says that chord roots are not just a theoretical aid but aware perceptible listening sensations and that they are equally in their being to virtual pitches (Terhardt, 1982, p. 23). To determine the virtual pitch of a harmonic complex tone Terhardt develops the following – here strongly combined – algorithmic method (following ibid., p. 37 f.): The signal is split into its partials, where masked partials are excluded. The following is a weighting of the partials based on partial maskings and the dominant spectral region and finally the determination of the virtual pitch by subharmonic coincidence detection (from ibid., p. 37): As Terhardt says, here each relevant partial is under investigation, if there is a fundamental pitch to which he stands in a harmonic overtone ratio. He further says that the ear uses the partials till the 10th partial as system parameters and so the search for the subharmonic root of every partial includes partials 1 to 10 (following ibid.). The results are compared and virtual pitches with different salience can

be determined. Terhardt applies an algorithm for the prediction of chord roots by investigating the individual chord tones in terms of their relationships in the overtone scale to possible root candidates and compares the possibilities. Thus he elicits possible roots n-th degree – these are not necessarily part of the chord.

<i>Akkordtöne:</i>	<i>c</i>	<i>f</i>	<i>a</i>
unison	C	F	A
-1 Quint	F	B ^b	D
-1 gr. Terz	As	Des	F
+1 Ganzton	D	G	H
-1 Ganzon	B	Es	G

<i>Grundtöne</i>	3. Grades: F
	2. Grades: D, B ^b , G

Fig. 11: Predicting the root of F major in second inversion following Ernst Terhardt (Terhardt, 1982, p. 41).

The intervals (perfect) unison, (perfect) fifth downwards, major third downwards, whole step upwards and whole step downwards are the (equal tempered) distances of the reversed overtone scale (always relating to the first partial) and in simplified and summarized form (the octave downwards as second partial is therefore not given in addition, since it is already contained in the unison and so on). In Figure 11 the intervals are called "unison" for unison, "Quint" for fifth, "Terz" for third and "Ganzton" for whole step (following Terhardt, 1982, p. 41).

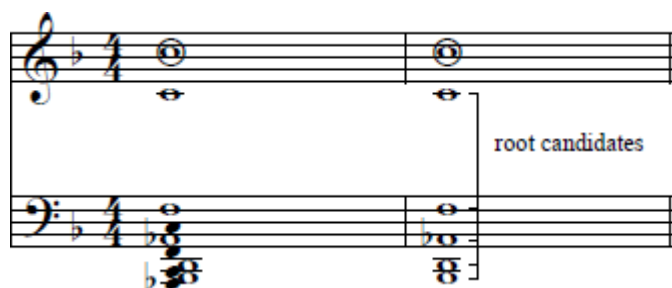


Fig. 12: Determination of the root candidates (following Terhardt, 1982) – encircled notes: chord tone, quarter notes: summarized tones, whole notes (not encircled): root candidates.

The problems here, however, are that on the one hand Terhardt's comparison of harmonic complex tones with chords is not scientifically verifiable, and on the other hand that the application of the overtone scale on chords, which consist of notes of the equal and not of the natural temperament, must be questioned. Terhardt also applies his algorithm only to simple examples, but neither to complex chord structures, nor to chord-chains.

Although Parncutt builds on Terhardt's model he certifies to this, however, to deviate in part too much from absolutely plausible patterns of explanation in music theory – for example, at the minor chord (cf. Parncutt, 1988, p. 65). Parncutt bases on the premise that different roots can be perceived with one chord, depending on the pitch classes it has, in which harmonic context it occurs and how it is layered (i.e., which sound in the bass is, etc.) (ibid., p. 87). He extends Terhardt's model and performs a more difficult weighting of both the so-called "root supports" (i.e. the root candidates) and the major and minor scales, which he designates by the term "prevailing tonality" (Parncutt, 1997, p. 185 f.). And he implements that into his model: "According to Terhardt (Terhardt, 1982) [Parncutt, 1988], the virtual pitch at the root of a chord is generated by the chord's tones and that the intervals octave/unison, perfect fifth, major third, minor seventh and major second/ninth determine the root. I call these intervals *root supports* (see Table 2; Parncutt 1988) [Parncutt, 1988]." (Parncutt, 2009, p. 128). Parncutt further says: "The chord-root model includes free parameters called *root-support weights*. These are quantitative estimates of the influence of each root-support interval on the salience of the virtual pitch at the lower tone of the interval, and hence on the perceived root of a chord." (ibid.). Regarding root ambiguity Parncutt distinguishes between the perception of different roots in different harmonic contexts and the perception of different roots at the same time: "A profile with two main peaks may cause a listener to perceive one peak or the other at different times (ambiguity), or both at once (multiplicity)." (ibid., p. 134, footnote 7).

A detailed explanation of Parncutt's algorithms for calculating root candidates would go by far beyond the scope of this talk and also can be read at Goldbach (Goldbach, 2009). What one can accuse partly to Parncutt is, according to Goldbach (cf. *ibid.*, p. 401), that he estimates the weighting of the minor third, which he introduces as an additional interval for root prediction (cf. Parncutt, 1988, p. 77). Moreover, here the comparison of the constitution of a chord with that of a complex tone is also difficult to prove and so to question as basis of a model for the prediction of chord roots. The model of Parncutt partly bases on Terhardt's research and also was taken up and expanded or assembled into analytical models by other theorists. However, there are also other approaches to the determination of basic chord tones – for instance of Balsach or Moraitis, who comes back to a model of Mackamul and differs between "primären" and "nominellen" (cf. Moraitis, 2006, p. 58) (that means between "primary" and "nominal") roots, and so on – but these are omitted here due to time constraints. Generally, however, major problems that occur with different models have to be highlighted: Some theorists derive their models from the overtone scale – e.g. also, as already mentioned, Rameau, who uses the occurrence of the major chord by the partials 4, 5 and 6 as its derivation. Hindemith, Terhardt and Parncutt, which use the combination tones as the basis of their models, also take the overtone scale as explanation.

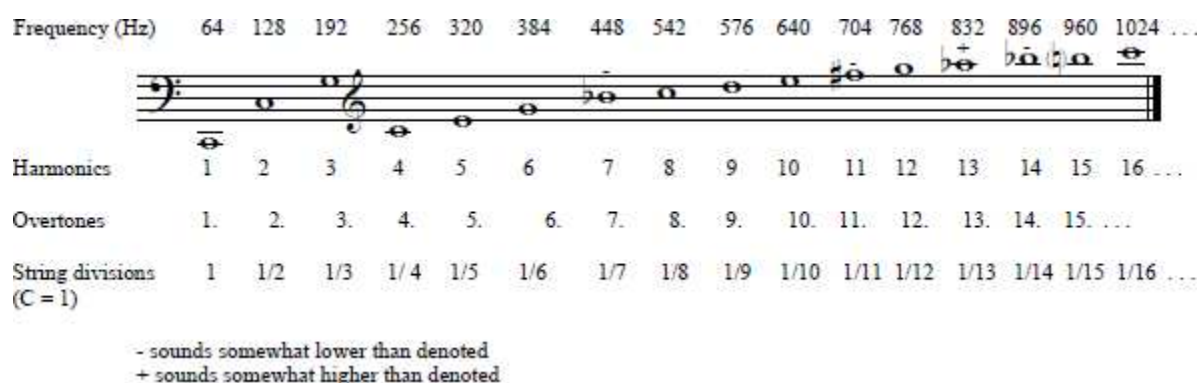


Fig. 13: Overtone scale, according to Paul Hindemith (Hindemith, 1940, p. 34).

This leads to two fundamental problems: Firstly, the overtone scale itself does not provide the complete material of the chromatic equal-tempered scale – (cf. Fig. 13) this anomaly is usually left out simple, but is a fundamental problem. Because of that, each model based on the overtone scale has to be constructed according to arbitrary selection procedures – e.g. by matching the deviating frequencies. Secondly, the overtone scale is an infinite range of multiples of a fundamental frequency. Thus the question of the limitation of the scale for the creation of a model rises. The occurrence of combination tones indeed has to be considered

in numerous acoustic phenomena; however the comparison of a chord with a harmonic complex tone must be questioned. For music theory relevant frequencies, mostly occurring in nature (those, which are produced by instruments and so on), activate in the majority of cases multiples of a fundamental frequency – these are the so called overtones. But the tones of a chord are, with exception of the major triad and also even the dominant seventh chord, rarely relevant multiples of a perceived root. Thus it is questionable whether combination tones or virtual pitches can be used for the prediction of the root of a chord. Because of the incompatibility of the overtone scale with the equal temperament and the possible fundamental difference between chords and single tones, each model comes ultimately to a point where the necessary premises do not base on a scientific observation, but rather on aesthetic motivated aspects. Since the human ear can in many cases perceive chord roots separated from aesthetic criteria and harmonic contexts, a theory of predicting chord roots should be based on highly scientific criteria. Hindemith partly tried this, but failed at the arbitrariness of his theory. On the other side, Terhardt and Parncutt were able to base their models for the prediction of chord roots already on mathematical operations and scientific premises – although the assumption that the essence of single tones is equally to that of chords still remains questionable.

It is very important for both a modern teaching of composition, as well as the investigation of historical works, to answer the question of predicting chord roots in the best possible manner. In this context especially music analysis has to consider the prediction of chord roots, as it is an attempt to discuss musical works, to explain their nature and to make connections visible within a composition as well as between different compositions. Terhardt writes in connection with his theory of virtual pitches as roots, that the order of roots that occurs in a tonal piece marks its harmonic process. Further that the root of sound represents its specific harmonic quality and that the harmonic structure of a tonal piece is highly marked, if the order of chord roots is in simple relations – particularly in fifth or fourth (following Terhardt, 1998, p. 398).

The prediction of chord roots also has applications in perceptual fields of music, such as music psychology and music therapy, because the logical explanation of relations between roots is the only tool to allow the study of the effects of music on the human psyche. Especially in medical areas new and more specific treatments could be developed and implemented through the integration of theoretical models of chord root prediction. Aspects of the doctrine of affections of the baroque period that were labeled as relatively superficial, gains new

relevance in modern analytical methods and the apparently stereotypical view of musical associations (such as major for joy, minor for sorrow, the tritone for death, danger or distress and so on) is the foundation of new hypotheses for a better understanding of music. However, first of all it is necessary to clarify key issues including determining the best approach for predicting the root of a chord – on the one hand to examine such analytical methods in a scientific manner and on the other hand to use them professionally and effectively in medical areas. In this context the prediction of chord roots plays a central role besides the doctrine of melody, because – as mentioned above – by means of root prediction it is possible to explain the general perception of chords, and thus make it usable.

Despite the publication of numerous models for the prediction of roots, this remains highly controversial and the topic is of much scientific debate. On the one hand this underlines the importance of the issue for the mentioned sub-areas of music theory and some other disciplines, and on the other hand shows that the search for a suitable model for the prediction of chord roots is not yet finished. A significant problem at the published models is as yet the lack of independent and interdisciplinary studies with representative numbers of participants. This is probably because there are not so many experts in this special subject who can bring both cross-curricular competencies, as well as detailed knowledge of the model to be examined. Furthermore, large scale studies require both a high level of time commitment and substantial funding. Relevant studies indeed were done by Houtsma/Goldstein or Parncutt, but there is still a lack of independent comparative values. However it is clear that since the establishment of psychoacoustics and their increased acceptance in music theory, a more intensive interest in possibilities of chord root prediction prevails. This brings a lively discussion with it.

As aesthetically oriented models are in most cases rather arbitrary, future solutions will have to be more strongly based on physically and psychoacoustically detectable principles and to this end especially medical disciplines such as neuroscience and cognitive psychology must play a larger role in the discourse. It requires the interdisciplinary cooperation of different scientists, to find useful and verifiable solutions which can be investigated in various studies. The question of whether a construction of models based on the overtone scale is purposeful, remains – as already mentioned – questionable. Purely aesthetic starting points have so far also brought no solution and nor have completely natural scientific observations resolved the question of predicting chord roots thus far. This does not mean that one of the mentioned

approaches could not provide an answer: In a narrower sense mainly natural scientific approaches should be envisaged, whereat usually an appropriate calculus lacks. Will a solution to the problem be found in the near future? It is hard to say. The fact is that over the past decades, various models have been developed to address the problem of predicting chord roots in very different ways and therefore a wide range of discussion remains. On this basis further research needs to be done, so that modern methods of music analysis and music therapy and thus modern tools for composition and the research on music perception can be developed. And I hope to contribute to this work with my research.

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